A Reexamination of the Subcarrier Demodulator Assembly Data Limiter Suppression Factor

L. Kuo and L. Webster
Network Operations Office

The data limiter suppression factor is an important parameter in determining Subcarrier Demodulator Assembly (SDA) performance in the presence of subcarrier phase jitter. A new mathematical model for this suppression factor is presented which, unlike previous models, allows for variable data symbol transitional probabilities and data filter time constant. Each of these quantities is examined for its effect on the data suppression factor. Finally, an example is presented which shows effects on SDA performance for data symbol transitional densities other than 50%.

I. Introduction

The Subcarrier Demodulator Assembly (SDA) models for use in the DSN were designed on the assumption that data have a transition probability of 50% and that $\tau_D/T_{SY} = 1/3$, where τ_D is the data filter time constant (see Fig. 1), and T_{SY} is the data symbol period. This assumption, however, is limited in data analysis since it occurs quite frequently that the symbol transition density is other than 50% and τ_D/T_{SY} ranges from approximately 1/5 to 1/2.

The suppression factor (α') is a very important parameter in determining demodulation performance. Since (α') varies as a function of data transitional densities, τ_D/T_{SY}, and ST_{SY}/N_0 (signal energy to noise spectral density ratio into the data filter), a study was made to determine the data suppression factor (α') as a function of these variables. This article also presents the effects on SDA degradation (symbol energy to noise spectral density out of SDA/symbol energy to noise spectral energy into SDA) as τ_D/T_{SY} and symbol transition density change.

Figure 1 is a functional block diagram (BLK III only) for the Subcarrier Demodulator Assembly. The input signal is an RF signal at the IF frequency of the receiver. The receiver phase tracks the received carrier and heterodynes it to the IF frequency at a fixed phase. The received signal contains telemetry data in the form of a binary waveform which biphase modulates a square-wave subcarrier. The modulated subcarrier, which is also a binary waveform, in turn modulates the carrier. The purpose of the Subcarrier Demodulator Assembly is to recover the original binary telemetry waveform by synchronously demodulating both the carrier and subcarrier. The receiver provides a reference signal at 10 MHz to demodulate the carrier. The reference signal, to demodulate the subcarrier, is provided by the demodulator itself, a portion of which acts as a phase-locked loop to track the subcarrier. Both demodulation processes take place in the upper channel of Fig. 1. The output of the upper channel is the recovered binary waveform which is sent to another part of the overall system for detection. The output waveform $m(t)$ is also filtered and limited to provide an estimate $\hat{m}(t)$ of the binary waveform (the recovered waveform is typically contaminated with noise and not strictly binary).

The term $m(t) \cdot \hat{m}(t)$ represents the data symbol stream $m(t)$ multiplied by an estimate $\hat{m}(t)$ of the symbol stream. $\hat{m}(t)$, the voltage at the output of the data hard limiter, represents the data symbol stream $m(t)$ with serrations due to Gaussian receiver noise plus a time delay at data transition due to the data filter time constant τ_D (see Fig. 2). The average value of $m(t) \cdot \hat{m}(t)$ over many digit periods is designated as (α'), the data suppression factor (Ref. 1):

$$\alpha' = \overline{m(t) \cdot \hat{m}(t)} = (\text{fraction of time } \hat{m}(t) \text{ agrees with } m(t) - \text{fraction of time } \hat{m}(t) \text{ disagrees with } m(t))$$

II. Mathematical Model

In formulating the model (see Fig. 3), consider a binary input signal $x(t)$ of the form

$$x(t) = \cdots + X_{-3}D^{-3} + X_{-2}D^{-2} + X_{-1}D^{-1} + X_0 + X_1 D + X_2 D^2 + X_3 D^3 + \cdots$$

where X_n ($n = \cdots, -2, -1, 0, 1, 2, \cdots$) are independent binary random variables assuming the value of V with probability P and the value of $-V$ with probability $(1 - P)$, and D is the delay operator of time T_{SY}. Let this signal be immersed in white Gaussian noise $n(t)$, which is zero mean and has a two-sided noise spectral density of $N_0/2$. The composite signal $z(t) = x(t) + n(t)$ is passed through a first order linear filter with transfer function $F(s) = 1/(1 + \tau_D S)$.

The output $y(t)$ is then hard limited to produce the signal $u(t)$ (called data estimate), i.e.,

$$u(t) = +1, \quad \text{if } y(t) \geq 0$$
$$u(t) = -1, \quad \text{if } y(t) < 0$$

The problem to be examined then is to find the average value of the product of the data and data estimate, namely,

$$\alpha' = E\{u(t) \cdot x(t)\} = E\{\hat{m}(t) \cdot m(t)\}$$

as a function of $R = V^2T/N_0 = ST_{SY}/N_0$, P, and T_{SY}/τ_D.

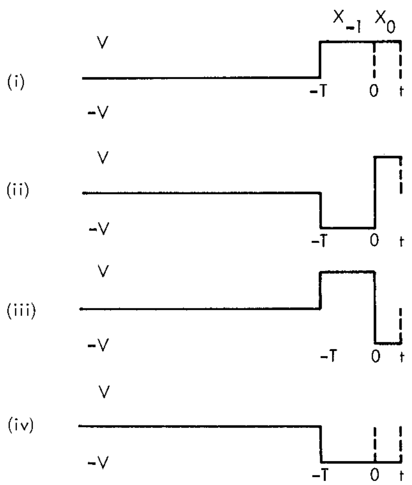
Since the filter has the greatest effect on most recent symbols of the incoming signal, the problem could be simplified by assuming that the symbol stream is all zero except the last two symbols and then adding small effects asserted by all previous symbols. In detail, initially assume that $x(t)$ takes on value V with probability P, and value $-V$ with probability $(1 - P)$ at time from 0 to t ($t < T$), $-T$ to 0, and has value 0 all the previous time ($-\infty$ to $-T$). Examine the filter output. The same process should be repeated by tracing back one more symbol period, namely, it takes on V, $-V$ with probability P, $1 - P$ at period $-2T$ to $-T$, 0 at period ($-\infty$ to $-2T$). Using the linear property of the filter, this result could be obtained from that of the initial case by adding small changes affected by this extra symbol period. Repeating this process by adding more symbols to be analyzed, the $\alpha' = E\{u(t) \cdot x(t)\}$ should finally converge to a limit, since less effects are being produced by the filter with each previous symbol added. The iteration process could be stopped when α' converges to a limit.

In obtaining the expected value of $u(t)\, x(t)$, it would be easier to assume a particular incoming waveform, find $E\{u_i(t)\, x_i(t)\}$ given that particular waveform (called conditional expected value given a particular case), then average the expected values over all the possible cases of incoming signals according to their probabilities of occurrences.

III. Mathematical Analysis

A. Analysis of Two-Symbol Periods (One Traced Back)

First, consider the case when one symbol period is being traced back (two symbols are being analyzed), namely, the incoming signal is assumed to have voltage zero from time $-\infty$ to $-T$, have voltage V with probability P, $-V$ with probability $(1 - P)$ at time $-T$ to 0, and have voltage V or $-V$ with probability P or $(1-P)$ from 0 to t $(t \leq T)$. In other words, one of the following four cases might occur:



q_i denotes the probability for each case to occur; then $q_1 = P^2$, $q_2 = (1-P)P$, $q_3 = P(1-P)$, $q_4 = (1-P)^2$, since X_n's are independent binary variables. Now, assuming one of the above four cases does occur, called i, then $\zeta_i(t) = E\{u_i(t)x_i(t)/i\}$. Since $u_i(t) = 1$ or -1, $\zeta_i(t)$ could be analyzed as in discrete cases with two sample points; thus,

$$\begin{aligned}
\zeta_i(t) &= E\{x_i(t)\cdot 1 \mid u_i(t) = 1\}\cdot P(u_i(t) = 1) \\
&\quad + E\{x_i(t)(-1) \mid u_i(t) = -1\}\cdot P(u_i(t) = -1) \\
&= X_{0_i}\cdot P(u_i(t) = 1) - X_{0_i}\cdot P(u_i(t) = -1) \\
&= X_{0_i}\cdot P(u_i(t) = 1) - X_{0_i}(1 - P(u_i(t) = 1)) \\
&= X_{0_i}(2P(u_i(t) = 1) - 1)
\end{aligned}$$

where X_{0i} indicates the magnitude of signal from time 0 to t at event i.

Now, analyze $P(u_i(t) = 1)$:

$$\begin{aligned}
P(u_i(t) = 1) &= P(y_i(t) \geq 0) \\
&= P\left(\int_0^\infty \frac{1}{\tau_D} e^{-\lambda/\tau_D} z(t-\lambda)\, d\lambda \geq 0/i\right) \\
&= P\left(\int_0^\infty e^{-\lambda/\tau_D}(x(t-\lambda) + n(t-\lambda))d\lambda \geq 0/i\right), \\
&\qquad \text{since } \tau_D > 0 \\
&= P\left(\int_0^\infty n(t-\lambda)\, e^{-\lambda/\tau_D}\, d\lambda \geq \right. \\
&\qquad \left. - \int_0^\infty e^{-\lambda/\tau_D} x_i(t-\lambda)\, d\lambda/i\right)
\end{aligned}$$

Now let

$$\begin{aligned}
f_i(t) &= \int_0^\infty e^{-\lambda/\tau_D} x_i(t-\lambda) d\lambda \\
&= \int_0^t e^{-\lambda/\tau_D} x_{0_i}\, d\lambda \\
&\quad + \int_t^{t+T} e^{-\lambda/\tau_D} x_{-1_i}\, d\lambda + 0
\end{aligned}$$

where x_{-1_i} indicates the magnitude of the signal from $-T$ to 0 at event i. Then solving for specific values of $f_i(t)$ yields:

$$\begin{aligned}
f_1(t) &= V\tau_D(1 - e^{-t/\tau_D}) + V\tau_D(e^{-t/\tau_D} - e^{-(t+T)/\tau_D}) \\
f_2(t) &= V\tau_D(1 - e^{-t/\tau_D}) - V\tau_D(e^{-t/\tau_D} - e^{-(t+T)/\tau_D}) \\
f_3(t) &= -V\tau_D(1 - e^{-t/\tau_D}) + V\tau_D(e^{-t/\tau_D} - e^{-(t+T)/\tau_D}) \\
f_4(t) &= -V\tau_D(1 - e^{-t/\tau_D}) - V\tau_D(e^{-t/\tau_D} - e^{-(t+T)/\tau_D})
\end{aligned}$$

Since

$$\int_0^\infty n(t-\lambda)\, e^{-\lambda/\tau_D}\, d\lambda$$

has Gaussian distribution with zero mean and $N_0\tau_D/4$ variance, then

$$P\left(\int_0^\infty n(t-\lambda)\, e^{-\lambda/\tau_D}\, d\lambda \geq -f_i(t)\right) = \frac{1}{2}\operatorname{erf}\left[f_i(t)\left(\frac{2}{N_0\tau_D}\right)^{1/2}\right] + \frac{1}{2}$$

This claim will be proved rigorously in Subsection C. At this point, it is observed that no closed-form expression could be obtained in evaluating $\operatorname{erf}[f_i(t)(2/N_0\tau_D)^{1/2}]$ as function of $R = V^2T/N_0$, P, and T_{SY}/τ_D. A computer program will be necessary to evaluate these functions. Now, average over all the conditional expected values to obtain $E[u(t)x(t)]$.

Conditional expected value for case i:

$$\zeta_i(t) = X_{0_i}[2P(u_i(t) = 1) - 1] = X_{0_i}\operatorname{erf}\left[f_i(t)\left(\frac{2}{N_0\tau_D}\right)^{1/2}\right]$$

$$h(t) = \Sigma_i\, q_i\, \zeta_i(t)$$

$$\begin{aligned}
\alpha' = E[u(t)x(t)] &= \frac{1}{T}\int_0^T h(t)\, dt \\
&= \Sigma_i\, q_i \frac{x_{0_i}}{T}\int_0^T \operatorname{erf}\left[f_i(t)\left(\frac{2}{N_0\tau_D}\right)^{1/2} dt\right]
\end{aligned} \tag{1}$$

We will now reformulate Eq. (1) in a more convenient form for computer solution:

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\left[f_1(t)\left(\frac{2}{N_0\tau_D}\right)^{1/2}\right] dt$$

$$= \frac{V}{T}\int_0^T \operatorname{erf}\left\{\left[V\tau_D\left(1 - e^{-t/\tau_D}\right) + V\tau_D\left(e^{-t/\tau_D} - e^{-(t+T)/\tau_D}\right)\right]\frac{\sqrt{2}}{\sqrt{N_0\tau_D}}\right\}$$

Let

$$R = \frac{V^2T}{N_0}, \qquad \lambda = \frac{T}{\tau_D}$$

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\left[\sqrt{\frac{V^2T}{N_0}}\cdot\frac{\sqrt{\tau_D}\sqrt{2}}{\sqrt{T}} \cdot\left(1 - e^{-t/\tau_D} + e^{-t/\tau_D} - e^{-(t+T)/\tau_D}\right)\right] dt$$

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\left\{\sqrt{2R\lambda^{-1}}\left[1 + e^{-t/\tau_D}\left(-1 + 1 - e^{-\lambda}\right)\right]\right\} dt$$

Now let

$$\alpha_1 = \sqrt{2R\lambda^{-1}}$$

$$\beta_1 = -1 + 1 - e^{-\lambda}$$

$$u = e^{-t/\tau_D}$$

Then

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\left(\alpha_1 + \alpha_1\beta_1 e^{-t/\tau_D}\right) dt$$

$$= \frac{V}{T}\int_1^{e^{-\lambda}} \operatorname{erf}\left(\alpha_1 + \alpha_1\beta_1 u\right)\frac{du}{u}\left(-\tau_D\right)$$

$$= \frac{V}{\lambda}\int_{e^{-\lambda}}^1 \frac{\operatorname{erf}\left(\alpha_1 + \alpha_1\beta_1 u\right)}{u} du$$

The same procedures are used to obtain ζ_2, ζ_3, ζ_4 such that:

$$\alpha = \sum_{i=1,4} q_i\zeta_i \tag{2}$$

where

$$\zeta_i = \frac{V}{\lambda}\int_{e^{-\lambda}}^1 \frac{\operatorname{erf}\left(\alpha_i + \alpha_i\beta_i u\right)}{u} du, \qquad i = 2,3,4 \tag{3}$$

$$\alpha_2 = \sqrt{2R\lambda^{-1}}$$

$$\alpha_3 = \alpha_4 = -\sqrt{2R\lambda^{-1}}$$

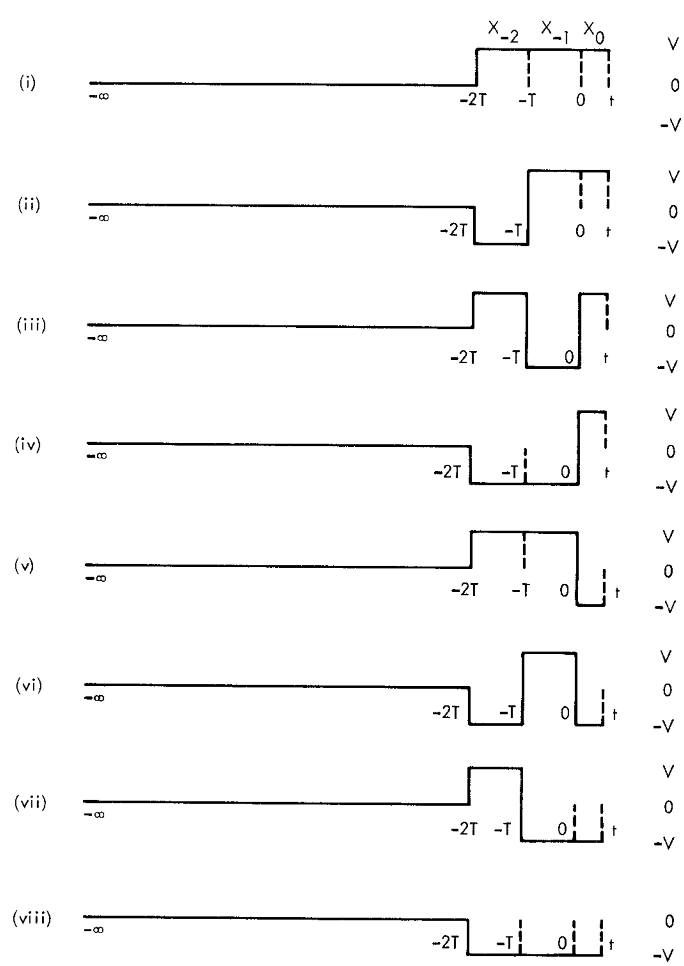
$$\beta_2 = -1 - 1 + e^{-\lambda}$$

$$\beta_3 = 1 + 1 - e^{-\lambda}$$

$$\beta_4 = 1 - 1 + e^{-\lambda}$$

B. Analysis of Three-Symbol Periods (Two Traced Back)

After analyzing the initial condition as in Subsection A, the next step is to add one more symbol to be traced back. It is necessary to average the following eight cases whose probabilities of occurrences are P^3, $(1-P)P^2$, $P(1-P)P$, $(1-P)^2P$, $P^2(1-P)$, $(1-P)P(1-P)$, $P(1-P)^2$, and $(1-P)^3$:



$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\left(f_1(t)\left(\frac{2}{N_0\tau_D}\right)^{1/2}\right) dt$$

$$f_1 = V\tau_D(1 - e^{-t/\tau_D}) + V\tau_D(e^{-t/\tau_D} - e^{-(t+T)/\tau_D}) + V\tau_D(e^{-(t+T)/\tau_D} - e^{-(t+2T)/\tau_D})$$

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\sqrt{\frac{V^2T}{N_0}}\frac{\sqrt{\tau_D}\sqrt{2}}{\sqrt{T}} \cdot (1 - e^{-t/\tau_D} + e^{-t/\tau_D} - e^{-(t+T)/\tau_D} + e^{-(t+T)/\tau_D} - e^{-(t+2T)/\tau_D})\, dt$$

Letting $T/\tau_D = \lambda$,

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}\{\sqrt{2R\lambda^{-1}}\,[1 + e^{-t/\tau_D} \cdot (-1 + 1 - e^{-\lambda} + e^{-\lambda} - e^{-2\lambda})]\}\, dt$$

Let

$$\alpha_1 = \sqrt{2R\lambda^{-1}}$$

$$\beta_1 = -1 + 1 - e^{-\lambda} + e^{-\lambda} - e^{-2\lambda}$$

$$u = e^{-t/\tau_D}$$

Then,

$$\zeta_1 = \frac{V}{T}\int_0^T \operatorname{erf}(\alpha_1 + \alpha_1\beta_1 e^{-t/\tau_D})\, dt$$

$$= \frac{V}{T}\int_1^{e^{-\lambda}} \operatorname{erf}(\alpha_1 + \alpha_1\beta_1 u)\frac{du}{u}(-\tau_D)$$

$$= \frac{V}{\lambda}\int_{e^{-\lambda}}^1 \frac{\operatorname{erf}(\alpha_1 + \alpha_1\beta_1 u)}{u}\, du$$

The same procedures are applicable to obtain

$$\zeta_i = \frac{V}{\lambda}\int_{e^{-\lambda}}^1 \frac{\operatorname{erf}(\alpha_i + \alpha_i\beta_i u)}{u}\, du, \qquad i = 2,3,\cdots,8 \tag{4}$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \sqrt{2R\lambda^{-1}}$$

$$\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = -\sqrt{2R\lambda^{-1}}$$

$$\beta_2 = -1 + (1 - e^{-\lambda}) - (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_3 = -1 - (1 - e^{-\lambda}) + (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_4 = -1 - (1 - e^{-\lambda}) - (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_5 = 1 + (1 - e^{-\lambda}) + (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_6 = 1 + (1 - e^{-\lambda}) - (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_7 = 1 - (1 - e^{-\lambda}) + (e^{-\lambda} - e^{-2\lambda})$$

$$\beta_8 = 1 - (1 - e^{-\lambda}) - (e^{-\lambda} - e^{-2\lambda})$$

Continue this iteration process by adding more symbols to be analyzed. A computer program was written which shows $E[u(t)\,x(t)]$ converges very rapidly. Thus, an accurate solution could be obtained with the two or three symbols traced back.

C. Proof of the Claim

Statement of the claim:

$$\int_0^\infty n(t-\lambda)\, e^{-\lambda/t_D}\, d\lambda$$

has Gaussian distribution with zero mean and $N_0\tau_D/4$ variance knowing that $E\{n(t)\} = 0$ and autocorrelation function of $n(t) = (N_0/2)\delta(\tau)$.

Proof:

$$E\left[\int_0^\infty n(t-\lambda)\, e^{-\lambda/\tau_D}\, d\lambda\right]$$

$$= \int_0^\infty E[e^{-\lambda/\tau_D}\, n(t-\lambda)]\, d\lambda$$

$$= \int_0^\infty e^{-\lambda/\tau_D}\, E[n(t-\lambda)]\, d\lambda$$

$$= \int_0^\infty e^{-\lambda/\tau_D} \cdot 0 d\lambda = 0$$

Variance of

$$\left[\int_0^\infty n(t-\lambda)\, e^{-\lambda/\tau_D}\, d\lambda\right]$$

$$= E\left[\int_0^\infty e^{-\lambda/\tau_D}\, n(t-\lambda)\, d\lambda \int_0^\infty e^{-\beta/\tau_D}\, n(t-\beta)\, d\beta\right]$$

$$= \int_0^\infty\int_0^\infty e^{-\lambda/\tau_D}\, e^{-\beta/\tau_D}\, E[n(t-\lambda)\, n(t-\beta)]\, d\lambda d\beta$$

$$= \int_0^\infty \int_0^\infty e^{-\lambda/\tau_D} e^{-\beta/\tau_D} \cdot \frac{N_0}{2} \delta(\beta - \lambda)\, d\lambda d\beta$$

$$= \int_0^\infty e^{-\beta/\tau_D} \left[\int_0^\infty e^{-\lambda/\tau_D} \frac{N_0}{2} \delta(\beta - \lambda)\, d\lambda \right] d\beta$$

$$= \int_0^\infty e^{-\beta/\tau_D} \left(\frac{N_0}{2} e^{-\beta/\tau_D} \right) d\beta$$

$$= \frac{N_0}{2} \frac{\tau_D}{2} e^{-2\beta/\tau_D} \Big|_0^\infty = \frac{N_0 \tau_D}{4}$$

so

$$n(t) = \int_0^\infty e^{-\lambda/\tau_D} n(t - \lambda)\, d\lambda \qquad \epsilon\, n\left(0, \frac{N_0 \tau_D}{4}\right)$$

D. Find Probability That $n(t) \geq -f_i(t)$

$$P(n(t) \geq -f_i(t)) = \int_{-f_i(t)}^{\infty} \frac{e^{-x^2} \Big/ \left(2 \cdot \frac{N_0 \tau_D}{4}\right)}{\sqrt{2\pi \frac{N_0 \tau_D}{4}}}\, dx$$

$$= \int_{-f_i(t)}^{\infty} \frac{e^{-x^2} \Big/ \left(\frac{N_0 \tau_D}{2}\right)}{\sqrt{\frac{\pi N_0 \tau_D}{2}}}\, dx - (A)$$

Let

$$y^2 = \frac{X^2}{\frac{N_0 \tau_D}{2}}$$

$$y = \sqrt{\frac{2}{N_0 \tau_D}}\, x$$

$$dy = \sqrt{\frac{2}{N_0 \tau_D}}\, dx$$

$$(A) = \int_{-f_i(t)(2/N_0\tau_D)^{1/2}}^{\infty} e^{-y^2} \cdot \sqrt{\frac{2}{N_0 \tau_D}} \cdot \sqrt{\frac{\pi N_0 \tau_D}{2}}\, dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{-f_i(t)(2/N_0\tau_D)^{1/2}}^{\infty} e^{-y^2} dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{-f_i(t)(2/N_0\tau_D)^{1/2}}^{0} e^{-y^2} dy + \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-y^2} dy$$

$$= \frac{1}{2} \operatorname{erf}\left\{ [f_i(t)] \left(\frac{2}{N_0 \tau_D} \right)^{1/2} \right\} + \frac{1}{2}$$

IV. Results/Discussion

Using the iteration process discussed in Subsections III-A and III-B, we obtain the data limiter suppression factor averaging over three symbols as follows:

$$\alpha' = E\{u(t) \cdot x(t)\} = \sum_{i=1}^{16} q_i \zeta_i \tag{5}$$

where

q_i = probability of the ith event

$$\zeta_i(t) = \frac{V}{\lambda} \int_{1-\lambda}^{1} \frac{\operatorname{erf}(\alpha_i + \alpha_i \beta_i u)}{u}\, du \tag{6}$$

$\lambda = T_{SY}/\tau_D$

$V = 1$

U = variable of integration

$q_1 = P^4$; $q_2 = P^3(1 - P)$; $q_3 = q_2$; $q_4 = P^2(1 - P)^2$;

$q_5 = q_2$; $q_6 = q_4$; $q_7 = q_4$; $q_8 = (1 - P)^3 P$; $q_9 = -q_2$;

$q_{10} = -q_4$; $q_{11} = -q_4$; $q_{12} = -q_8$; $q_{13} = -q_4$;

$q_{14} = -q_8$; $q_{15} = -q_8$; $q_{16} = (1 - P)^4$

where

P = data transition probability

α_1 thru $\alpha_8 = \sqrt{2R\lambda^{-1}}$

α_9 thru $\alpha_{16} = -\sqrt{2R\lambda^{-1}}$

$\beta_1 = [-1 + D1 + D2 + D3] \cdot A$

$\beta_2 = [-1 + D1 + D2 - D3] \cdot A$

$\beta_3 = [-1 + D1 - D2 + D3] \cdot A$

$\beta_4 = [-1 + D1 - D2 - D3] \cdot A$

$\beta_5 = [-1 - D1 + D2 + D3] \cdot A$

$\beta_6 = [-1 - D1 + D2 - D3] \cdot A$

$\beta_7 = [-1 - D1 - D2 + D3] \cdot A$

$\beta_8 = [-1 - D1 - D2 - D3] \cdot A$

$\beta_9 = [1 + D1 + D2 + D3] \cdot A$

$\beta_{10} = [1 + D1 + D2 - D3] \cdot A$

$\beta_{11} = [1 + D1 - D2 + D3] \cdot A$

$\beta_{12} = [1 + D1 - D2 - D3] \cdot A$

$\beta_{13} = [1 - D1 + D2 + D3] \cdot A$

$\beta_{14} = [1 - D1 + D2 - D3] \cdot A$

$$\beta_{15} = [1 - D1 - D2 + D3] \cdot A$$

$$\beta_{16} = [1 - D1 - D2 - D3] \cdot A$$

where

$$A = \sqrt{2R\lambda^{-1}}$$

$$D1 = 1 - \mathrm{EXP}(-\lambda)$$

$$D2 = \mathrm{EXP}(-\lambda) - \mathrm{EXP}(-2\lambda)$$

$$D3 = \mathrm{EXP}(-2\lambda) - \mathrm{EXP}(-3\lambda)$$

The validity of averaging over only three symbols to calculate the data suppression was investigated with a computer program which calculated the α' averaging over one symbol, two, or three successive symbols. As can be seen from Table 1a, the result converges rapidly for averaging over three symbols when T_{SY}/τ_D is 3. Although not presented, a similar test was made for T_{SY}/τ_D in the range 3 through 12 and probability of transitions over the entire range possible. Over these values, the data suppression factor as expressed in Eqs. (5) and (6) quickly converges, thus averaging over three symbols is sufficient. It should be noted that the results of Eqs. (5) and (6) become less accurate for T_{SY}/τ_D values of less than 3 due to greater filter "memory."

The effect of varying the T_{SY}/τ_D ratio for a 50% probability of data transition is shown in Table 1b and plotted on Fig. 4 for varying values of the data filter input symbol energy to noise spectral density (ST_{SY}/N_0). The data limiter suppression factor as plotted in Fig. 4 compares quite well with results published in Ref. 1.

Variations in α' as a function of transitional probabilities (P) are shown in Fig. 5. We see that pronounced changes in the data suppression factor for a constant T_{SY}/τ_D ratio and signal-to-noise ratio occur at very low (20%) or very high (80%) values of the transition probability. Since the Mariner Jupiter/Saturn (MJS) mission will be using data rates with transitional probabilities of 30 to 80%, this result is clearly important.

Finally, Subcarrier Demodulator Assembly (SDA) degradation for various T_{SY}/τ_D ratios occurring with BLK III/IV SDA designs (see Tables 2 and 3), transitional probabilities, and a fixed ST_{SY}/N_0 of 10 dB is examined using an SDA degradation model developed by Lesh (Ref. 2). These results are shown in Fig. 6. A symbol rate of 8.33 was used with a wide (BLK IV) SDA bandwidth. SDA degradation changes of greater than 0.1 dB can result for varying probabilities of transition.

Acknowledgment

The approach used was suggested by James Lesh.

References

1. Brockman, M. H., "An Efficient and Versatile Telemetry Subcarrier Demodulation Technique for Deep Space Telecommunication," presented at the 4th Hawaii International Conference on System Science, Jan. 1971.
2. Lesh, J., "A Re-Examination of Subcarrier Demodulation Performance," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. XVII, pp. 137–144, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1973.

Table 1. Table of suppression factor $E(u(t)x(t))$

Number of symbols being traced back	P	ST_{SY}/N_0, dB			
		−5	0	5	10
(a) $\lambda = T_{SY}/\tau_D = 3$					
1	0 or 1	0.47723	0.74420	0.95665	0.99967
2	0 or 1	0.48355	0.75142	0.95981	0.99974
3	0 or 1	0.48386	0.75177	0.95996	0.99974
1	0.5	0.33380	0.52830	0.70280	0.76276
2	0.5	0.33379	0.52827	0.70279	0.76285
3	0.5	0.33379	0.52827	0.70279	0.76285
1	0.2	0.38543	0.60603	0.79419	0.84804
2	0.2	0.38770	0.60860	0.79531	0.84813
3	0.2	0.38782	0.60873	0.79537	0.84813
(b) $\lambda = 6$					
3	0 or 1	0.35387	0.58578	0.85349	0.99018
3	0.5	0.29572	0.49181	0.72559	0.86038
3	0.2	0.31665	0.52564	0.77163	0.90711
(c) $\lambda = 12$					
3	0 or 1	0.25457	0.43630	0.69543	0.93211
3	0.5	0.23348	0.40060	0.64061	0.86539
3	0.2	0.24107	0.41345	0.66035	0.88941

Table 2. Data symbol rate selection (BLK III)

	$1/T_{SY}$, symbols/s	B_{IF}(data), Hz	G_{IF}(dist), dB	G_{IF}(WB), dB	T_{det}, ms	T_{out}, ms
1	Future	Blank	7	44	Blank	Blank
2	5.6–12	500	14	37	39	1800
3	12–27	500	14	37	18	820
4	27–56	500	14	37	8.1	390
5	56–120	5000	20	31	3.9	180
6	120–270	5000	20	31	1.8	82
7	270–560	5000	20	31	0.81	39
8	560–1200	50K	27	24	0.39	18
9	1200–2700	50K	27	24	0.18	8.2
10	2700–5600	50K	27	24	0.081	3.9
11	5600–12K	500K	35	16	0.039	
12	12K–27K	500K	35	16	0.018	
13	27K–56K	500K	35	16	0.0081	
14	56K–120K	3M	45	6	0.0039	
15	120K–270K	3M	45	6	0.0018	

Table 3. Data symbol rate selection (BLK IV)

	$1/T_{SY}$, symbols/s	Quad gen gain, dB	Select BW filter		τ_{det}, ms	τ_{out}, ms
			Gain, dB	BW_{IF}, kHz		
1	5.6–11.9	11	36	1.03	47.5	1800
2	12 –26.9	11	36	1.03	22.1	820
3	27 –55.9	11	36	1.03	10.0	390
4	56 –99.9	11	36	1.03	4.75	180
5	100 –219	22	25	11.9	2.21	180
6	220 –479	22	25	11.9	1.00	180
7	480 –999	22	25	11.9	0.475	180
8	1.00K–2.19K	29	18	49.5	0.221	180
9	2.20K–4.79K	29	18	49.5	0.100	180
10	4.80K–9.99K	36	11	209	0.0475	180
11	10.0K–21.9K	36	11	209	0.0221	180
12	22.0K–47.9K	42	5	1020	0.0100	180
13	48.0K–99.9K	42	5	1020	0.00475	180
14	100.0K–219.K	47	0	5000	0.00221	180
15	220.K–500.K	47	0	5000	0.00100	180

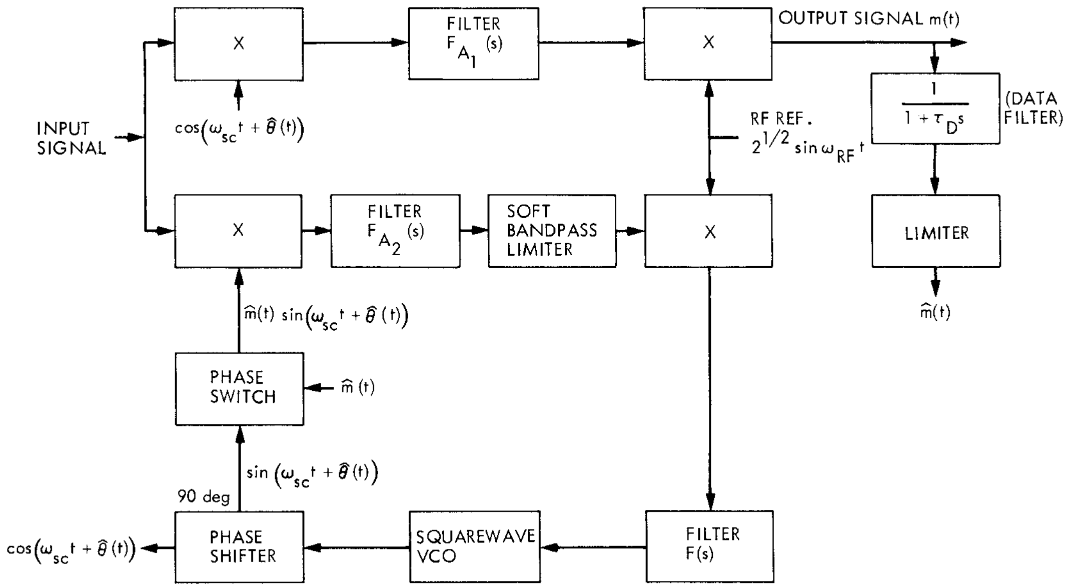


Fig. 1. Functional block diagram of telemetry subcarrier demodulator (BLK III)

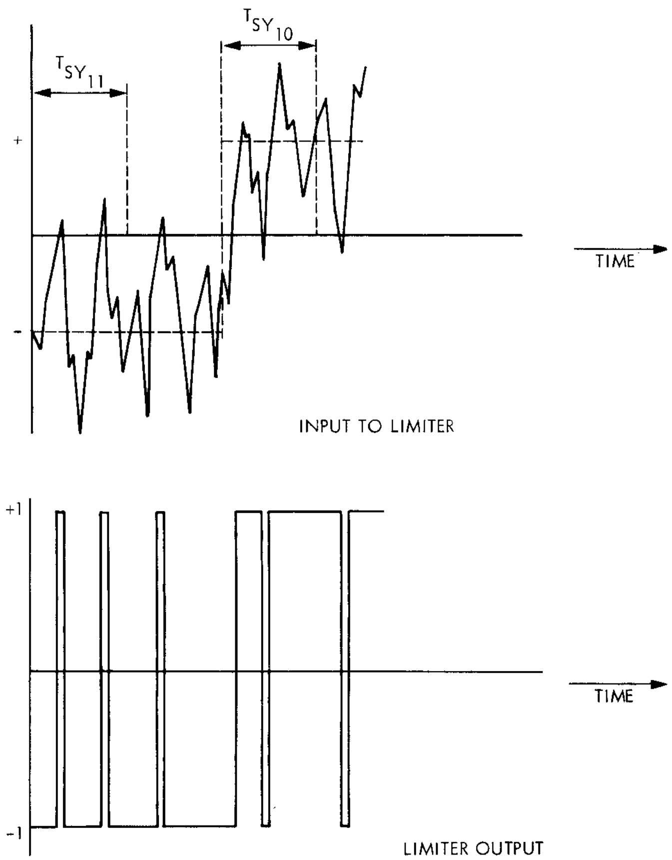


Fig. 2. Limiter input and output waveforms

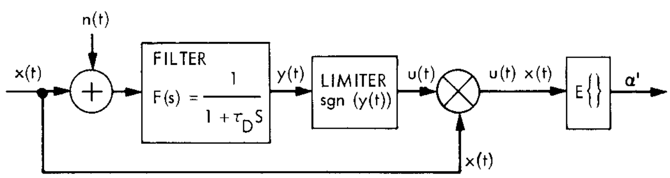


Fig. 3. Model of problem being examined

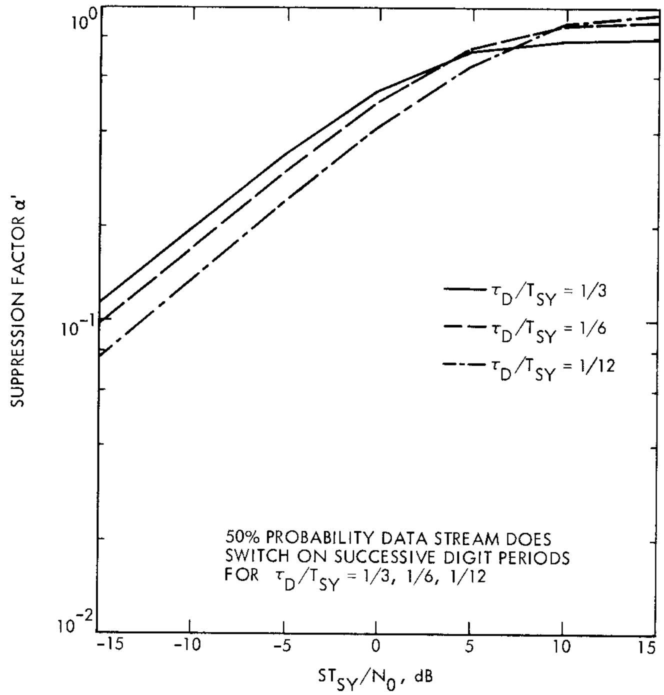


Fig. 4. MMTS subcarrier demodulator suppression factor vs ratio of signal energy per symbol to noise spectral density

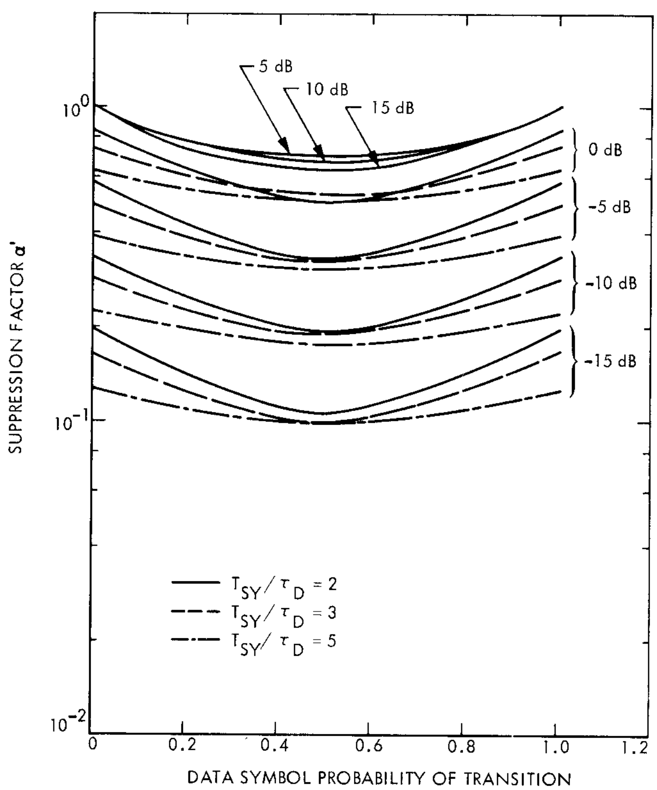


Fig. 5. Data suppression factor vs data symbol probability of transition

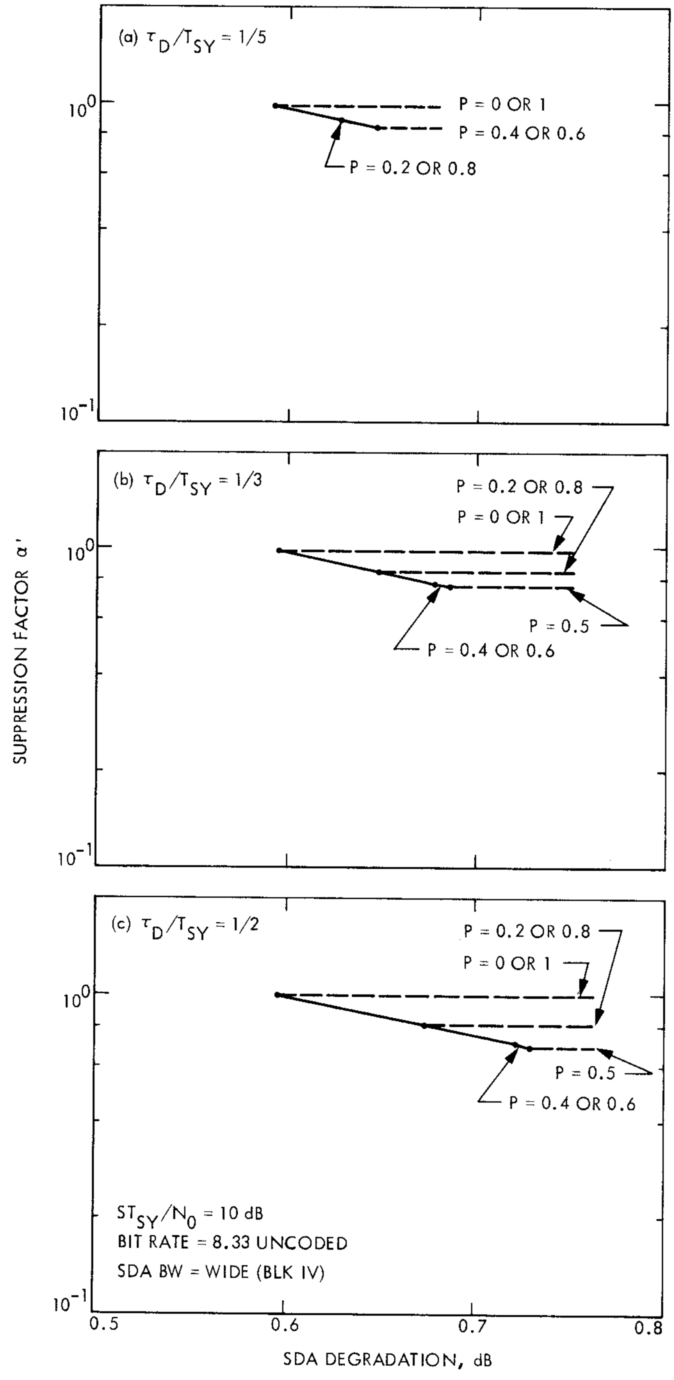
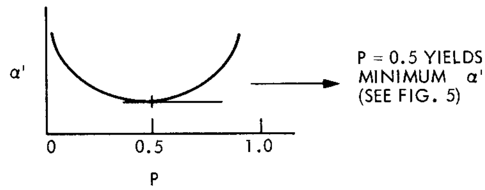


Fig. 6. BLK IV SDA model sensitivity to α' suppression factor and τ_D/T_{SY} variations

Appendix

The following analysis yields the data suppression factor α' for the special cases where the incoming signal is all 1 or all -1. Suppression factors thus obtained should agree with those of probability of transition equal to 0 or probability of transition equal to 1.



Assume incoming signal is all -1:

$$
\begin{aligned}
E(u(t)\,x(t)) &= -VP\{u(t)=1\} + (-V)(-1)P\{u(t)=-1\} \\
&= -VP\{u(t)=1\} + V[1 - P\{u(t)=1\}] \\
&= -V[2P\{u(t)=1\} - 1]
\end{aligned}
$$

$$
\begin{aligned}
P\{u(t)=1\} &= P\{y(t) \geq 0\} \\
&= P\left\{\int_0^\infty \frac{1}{\tau_D} e^{-\lambda/\tau_D}[x(t-\lambda) + n(t-\lambda)]\,d\lambda \geq 0\right\} \\
&= P\left\{\int_0^\infty e^{-\lambda/\tau_D} n(t-\lambda)\,d\lambda \geq -\int_0^\infty e^{-\lambda/\tau_D}(-V)\,d\lambda\right\} \text{ (since } \tau_D \geq 0\text{, incoming signal is } -V) \\
&= P\left\{\int_0^\infty e^{-\lambda/\tau_D} n(t-\lambda)\,d\lambda \geq V\tau_D\right\}
\end{aligned}
$$

Since

$$
n(t) = \int_0^\infty e^{-\lambda/\tau_D} n(t-\lambda)\,d\lambda \,\epsilon\, n\left(0, \frac{N_0\tau_D}{4}\right)
$$

so

$$
P\{n(t) \geq V\tau_D\} = \int_{V\tau_D}^\infty \frac{e^{-x^2}\Big/\left(2\dfrac{N_0\tau_D}{4}\right)}{\sqrt{2\pi\dfrac{N_0\tau_D}{4}}}\,dx - (B)
$$

Let

$$
y^2 = \frac{X^2}{\dfrac{N_0\tau_D}{2}}
$$

$$y = \sqrt{\frac{2}{N_0\tau_D}}\, x$$

$$dy = \sqrt{\frac{2}{N_0\tau_D}}\, dx$$

$$(B) = \int_{V\tau_D(2/N_0\tau_D)^{1/2}}^{\infty} \frac{e^{-y^2}\sqrt{2}}{\sqrt{\pi N_0\tau_D}}\, dy \sqrt{\frac{N_0\tau_D}{2}}$$

$$= \frac{1}{\sqrt{\pi}} \int_{V(2\tau_D/N_0)^{1/2}}^{\infty} e^{-y^2}\, dy$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-y^2}\, dy - \frac{2}{\sqrt{\pi}} \int_0^{V(2\tau_D/N_0)^{1/2}} e^{-y^2}\, dy\right)$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}\left(V\sqrt{\frac{2\tau_D}{N_0}}\right)\right]$$

so

$$E(u(t)\,x(t)) = -V\left\{2 \cdot \frac{1}{2}\left[1 - \mathrm{erf}\left(V\sqrt{\frac{2\tau_D}{N_0}}\right)\right] - 1\right\}$$

$$= V\,\mathrm{erf}\left(V\sqrt{\frac{2\tau_D}{N_0}}\right)$$

$$= V\,\mathrm{erf}\left(V\sqrt{\frac{V^2 T}{N_0}} \cdot \frac{\sqrt{2\tau_D}}{V\sqrt{T}}\right); \qquad \text{let } \lambda = \frac{T}{\tau_D}$$

$$= V\,\mathrm{erf}\left(\sqrt{\frac{2R}{\lambda}}\right)$$